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CONAN: CONUNCTION ANALYSIS TOOL FOR EU SST CA SERVICE USERS

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ABSTRACT

Over the last few years, both the space debris population and the number of active satellites in orbit has dramatically increased. The risk of collision for satellite missions is a problem more and more targeted thoroughly by all agents involved in SSA. The solutions for the conjunction analysis (CA service) have evolved towards a service that is provided by specialized centres to satellite operators. The EU SST consortium provides this service through two Operations Centres behaving as hot-redundant of each other. One of these OCs is the S3TOC (Spanish Space Surveillance and Tracking Operations Centre), for which a new **Con**junction **An**alysis (CONAN) tool has been developed and is presented in this paper.

This tool provides CA users, a more flexible and accessible mean to analyse and evaluate the risks of any conjunction assessed by the EU SST.
It is also a powerful tool for S3TOC analysts allowing them to reanalyse a conjunction with a different configuration: using most updated ephemeris, introducing manoeuvers, with other set of physical or mathematical parameters, etc. It also allows to compute, visualize and test different avoidance manoeuvre solutions for one specific conjunction or a set of simultaneous conjunctions and acknowledge its effect on further conjunctions for that satellite. This information is later shared with users and can be tackled to their needs. Additionally, CONAN provides the means to compute the observed covariance by comparing successive orbit determinations of the same satellite, to obtain the actual uncertainty in position being a mean to detect possible errors in the orbit determination. Finally CONAN works as an aggregator of the information related to conjunctions coming from the EU SST and from 18SPCS, directly connecting to the interfaces from both entities, and provides multiple graphical information (including a 3D representation of the conjunction) and statistics related to risk assessment.

This paper starts by presenting CONAN as a valuable tool to the CA service provided by the EU SST consortium and describes the advantages that it offers to S3TOC analysts and to spacecraft operators subscribed to it. An overview of the

design of the tool is presented and the state-of-the-art algorithms implemented are summarized.

1. INTRODUCTION

In order to promote the development of a capacity in the field of space surveillance in Europe, in 2014 the European Union created (through decision no. 541/2014/EU of the European Parliament and of the Council of 16 April 2014) a space surveillance support framework, the purpose of which is to develop an independent SSA/SST capability in Europe through the EU SST initiative. Since then, the nascent national space surveillance capabilities in the countries that are part of the EU SST consortium (Germany, France, Italy and Spain since 2016; Poland, Romania and Portugal since 2019) have been federated in a coordinated manner, and the EU SatCen acts as a point of contact for the services provided by the EU SST consortium.

Three services are provided by the EU SST: Collision Avoidance (CA) Service, Fragmentation Service and Re-entry service. The users of the CA Service are European Spacecraft Owners and/or Operators (O/Os) and the service provision is delegated to the national centres from Spain, S3TOC (Spanish SST Operations Centre) and France, COO (Centre d'Orbitographie Opérationnel, forming part of CNES). Both centres act in a hot-redundancy concept, meaning that both centres compute all the products for all the satellites subscribed to de European service but only one, depending on the satellite, makes their outputs public through the EU SST Service Provision Portal (contribution of the EU SatCen). This assures a fast handover of the CA Service between both national OCs in case one of them has a problem; never occurred until now.

The CA Service provided by the EU SST is focussed in the generation of autonomous CDMs (Conjunction Data Messages) out of the orbital information from O/Os and the measurements from the EU SST Sensor Network, composed of telescopes, surveillance radars and laser stations from all the member states and shared between the national OCs thought the EU SST Database (contribution of GSAC, Germany); but it also includes the re-evaluation of the CDMs produced by the 18thSPCS (18th Space Control Squadron from the USA), what is called the middleman service.

CDTI (Centro para el Desarrollo Tecnológico Industrial) is responsible of the S3TOC and it is operated by GMV with Deimos, Hisdesat and ImmediaT as subcontractors. CONAN project is part of the evolution activities of S3TOC to provide a better and more efficient service to users, keeping it homogenised with the one provided by the French OC. CONAN will be made available during 2022 to the O/Os subscribed to the CA Service of the EU SST and assigned to the S3TOC as Nominal OC.

2. FUNCTIONALITIES

CONAN (standing for **CON**junctions **AN**alysis Tool) has a double objective. In one side it will be used by S3TOC analysts to improve the S3TOC capacities and response times to user requests. In the other hand, it will be provided to O/Os subscribed to the service, in order to offer them more flexibility and accessibility to evaluate the risks for their satellites facing a conjunction assessed by the EU SST. A set of well thought functionalities have been put together on it. They are listed in this chapter.

At the moment of CA service first configuration, a set of pre-defined thresholds are agreed between O/O and OC on the variables defining a conjunction event. They define the so-called WARNING or IEs (Interest Event) and the ALERT or HIEs (High Interest Events).

During the service provision, the users upload their orbital information to the EU SST Service Provision Portal, which is retrieved by the CA Service OCs to compute CDMs which are publish back into the EU SST Service Provision Portal. All the ALERT CDMs launch also the computation of a map of possible Collision Avoidance Manoeuvers (CAMs) with different DVs and at different manoeuver times and the effect on terms of risk reduction. This information is also accessible thought the EU SST Service Provision Portal.

Whenever there is a particularly worrying conjunction, the O/O can share with the OC a “special ephemeris” which normally includes a potential avoidance manoeuvre, willing to check its effect on the conjunction. The OC re-computes the conjunction using this ephemeris and also launches a screening against the objects catalogue, to discard if the new orbit is causing new conjunctions or worsening any pre-existing one. The results are sent back to the O/O that decides whether or not to introduce the manoeuvre in the operations plan.

This process takes time and in most of the cases the final decision is not to manoeuvre. CONAN allows the O/Os to perform by their own basic analysis with potential manoeuvres, new ephemeris, satellite configuration changes or even different CA service configuration parameters (risk thresholds for instance), and have access to the re-computed CDMs and all the graphical information related to the computations. This improves and make more efficient, the communications between O/O and OC to deal with special cases.

2.1. CA SERVICE INFORMATION BOARD

CONAN connects to the EU SST Service Provision Portal to download the CDMs published by the OC and it also imports the CDMs from the 18th SPCS directly connecting to www.space-track.org. This allows to compare the solution from both sources in multiple ways:

- **CA Service summary panel**: to provide a fast overview of the status for the fleet, with particularly interesting information for each satellite or constellation such as the total number of CDMs from each source and for each risk level, the creation date of the oldest and the newest CDM, the time span covered by the last imported orbit, etc. This panel includes a timeline plot with one row per satellite and conjunction events represented by dots, coloured depending on the current risk level.

- **CDMs list panel**: One dedicated panel in tabular format to list and filter CDMs. The columns are configurable, sortable and filterable though the most important variables defining a CDM:

- Creation date,
- Source,
- Message ID,
- Conjunction event,
- Constellation,
- Orbital regime,
- Autonomy,
- Primary name,
- Primary Cospar Id,
- Primary Norad Id,
- Primary source,
- Primary ephemeris,
- Primary HBR,
- Secondary object type,
- Secondary name,
- Secondary Cospar Id,
- Secondary Norad Id,
- Secondary source,
- Secondary ephemeris,
- Secondary satcat size,
- Secondary HBR,
- Risk level,
- Miss distance,
- Radial distance,
- Along-track distance,
- Cross-track distance,
- PoC,
- Scaled PoC,
- Kp,
- Ks

The CDMs list panel includes also some the following graphical information:

- Conjunction B-plane
- Scaled PoC map of Kp-Ks
- Animated 3D view of the encounter

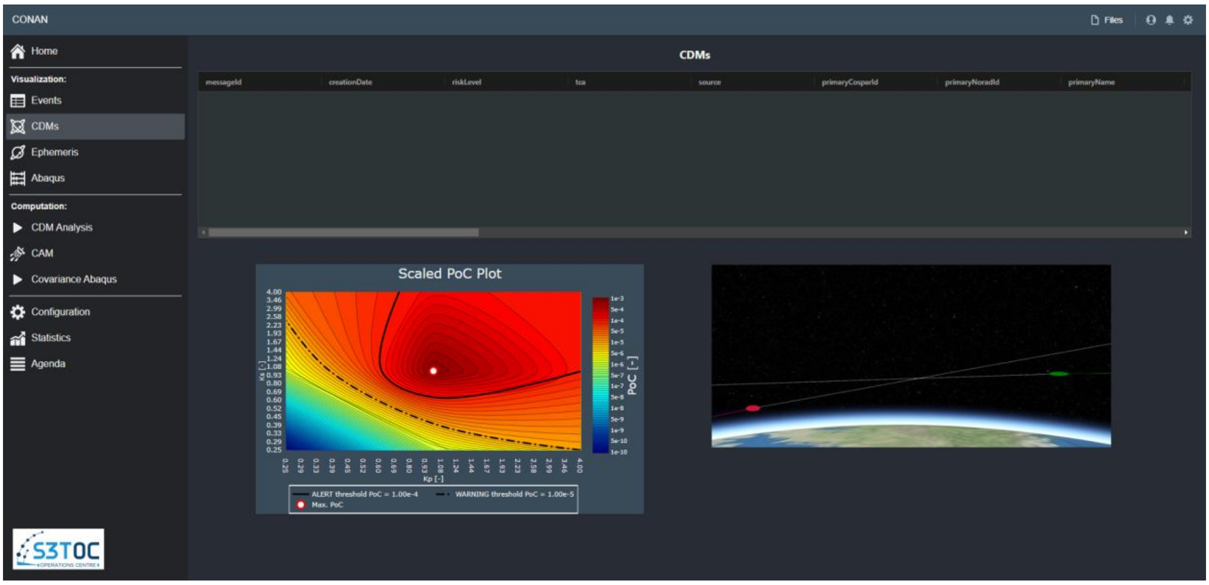

Figure 1 – CONAN CDMs list panel (development version)

- **Conjunction Events list panel**: Similarly to the CDMs list, a dedicated panel lists the conjunction events (one conjunction event is composed of multiple CDMs representing updates of the event). Again the columns are selectable, sortable and filterable through the following variables:
  - Constellation,
  - Orbital regime,
  - Primary name,
  - Primary COSPARID,
  - Primary Norad Id,
  - Secondary name,
  - Secondary object type,
  - Secondary satcat size,
  - Time since the last CDM,
  - Maximum reached ScPoC,
  - Minimum reached radial distance,
  - Minimum reached miss distance,

- Secondary Cospar Id,
- Secondary Norad Id,
- Risk level,
- Maximum reached risk level

The conjunctions events panel also includes the following plots:
- Miss distance evolution*,
- Radial distance evolution*,
- Cross distance evolution*,
- Tangential distance evolution*,
- PoC evolution*,
- Scaled PoC evolution*,
- Conjunction plane and covariance ellipse of its CDMs or a subset,

* All evolution plots show different series per type of CDM.

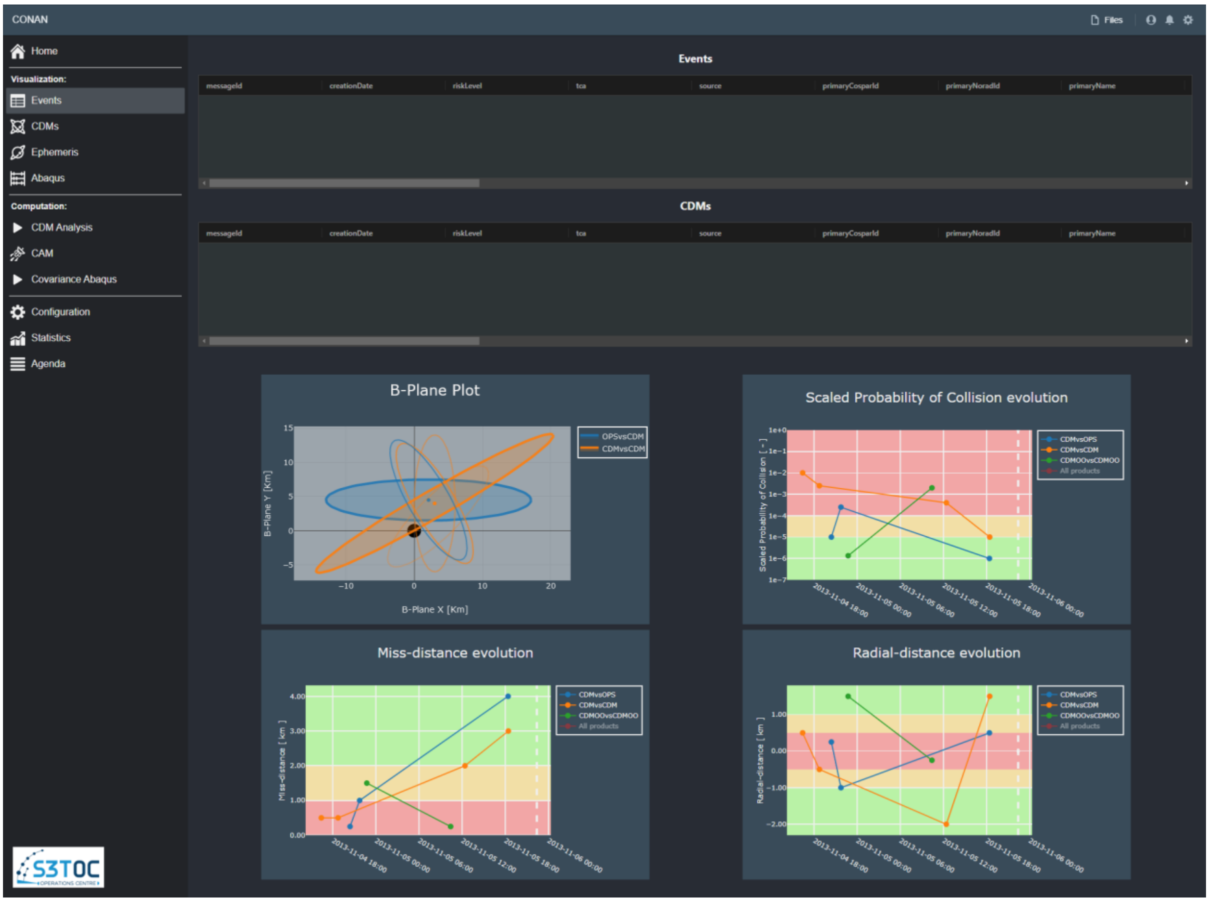

Figure 2 – CONAN Events Panel (development version)

Additionally in the Conjunction Events List Panel, the expected evolution of the probability of collision can be shown, according to the algorithm described in section 4.5.

2.2. CDM ANALYSIS

One of the main functionalities of CONAN is to reanalyse an existing CDM by modifying part of its configuration such as the orbital information of the involved objects or the algorithm for computation of probability of collision. The complete list of items that are modifiable to reanalyse a CDM is:

- Providing a different primary and/or secondary HBRs,

- Providing a different primary ephemeris,
- Manually introducing one or more manoeuvres (epoch, Delta V module and direction), or a CAM computation (see 2.3)
- Providing a different covariance for the primary manually introducing the covariance matrix at TCA or by providing a covariance abacus (see 2.4).
- Setting a different method for PoC (Akella-Alfriend / low relative velocity).
- Providing a different Kp and/or Ks. The software allows to choose a particular value, a fixed interval or to choose automatic computation of the optimum interval.
- Providing a different set of risk level thresholds.

The software allows to reanalyse more than one CDM in one execution with the selected configuration and the results are new CDMs stored in database and can be exported to the filesystem as CDMs in CCSDS-XML format.

Some of the parameters previously described can be preconfigured per satellite or per constellation, for instance a pre-configured set of risk level thresholds per constellation or a pre-configured HBR for each satellite. For those parameters the SW allows to choose the value from the input CDM, from configuration or manually introduce it for that run.

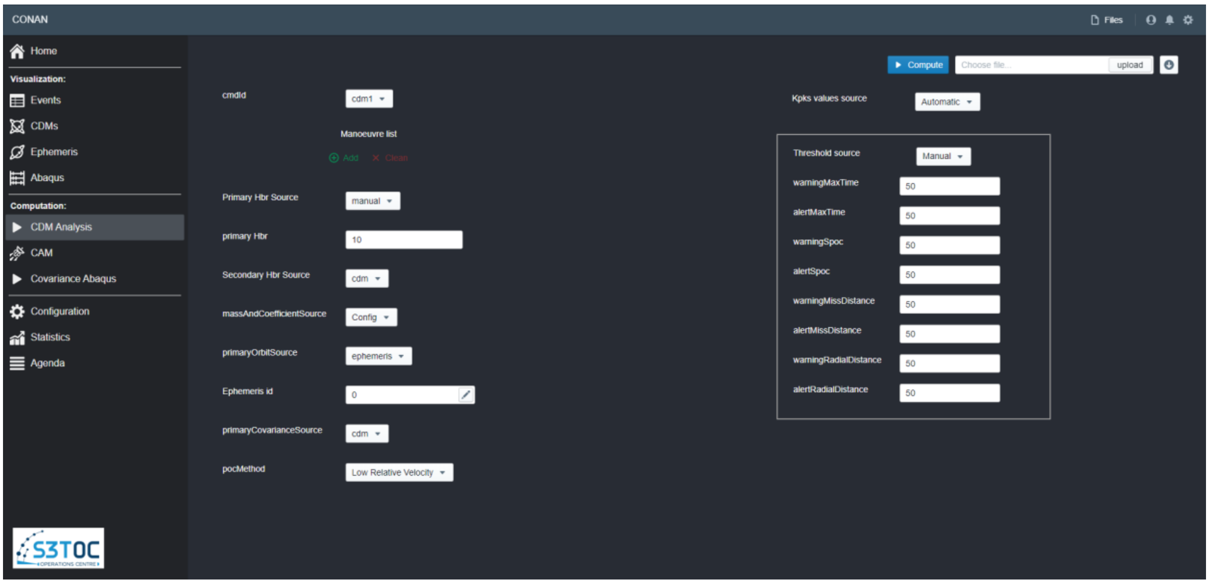

Figure 3 – CONAN CDM Analysis panel (development version)

2.3. CAM COMPUTATION

Another important functionality allows to compute a range of Collision Avoidance Manoeuvres (CAM) to avoid a particular conjunction event. Selected a CDM and choosing a particular configuration for CAM computation, provides a map of solutions (possible manoeuvres) to avoid or reduce the risk. The configuration of the panel allows to add the following constraints to the computation:

- Maximum allowed post CAM PoC.
- Minimum allowed post CAM miss distance.
- Minimum allowed post CAM radial clearance.

- Maximum available delta V or a fix delta V value.
- Time range before TCA to compute manoeuvres (in absolute time or in half orbital periods) or a fix manoeuvre epoch.

The output of the computation are three maps with the Delta V in the horizontal axis and the Miss Distance, the Radial Distance and the Scaled PoC in the vertical axes respectively, where each manoeuvre epoch is represented by a line (see Figure 19) The tool is also capable to provide the CAM solutions to reduce the global risk of more than one CDM (of different conjunction events) simultaneously.

The post CAM criteria (post CAM PoC, MD, RD) can be pre-configured per constellation or per satellite and the user can choose whether to use the values from configuration, or manually introduce them.

After running a manoeuvre computation, a specific CAM can be selected and all CDMs for the affected satellite with TCA after CAM are automatically reanalysed incorporating the calculated manoeuvre into the primary orbit. This allows to predict the effect of the CAM, not only in the initial CDM but in any other pre-existing conjunction event.

This module is automatically executed for each CDM imported in CONAN of HIE risk level, using the default configuration chosen for the satellite/constellation.

2.4. COVARIANCE ABACUS COMPUTATION

Sometimes, the covariance is not available in the primary orbit but it is a key variable for the computation of the probability of collision. In this cases it is useful to compute a covariance abacus also referred to as “observed covariance”. It is calculated out of the relative differences between successive uncorrelated orbit determinations (see algorithm description in 4.8). CONAN has a specific panel that allows to compute covariance abacus provided a list of ephemeris for the chosen satellite. Then those abacus can be used in the CDMs re-computation as a source for covariance information for the primary. CONAN also allows to export an OEM (Orbit Data Message) created from an input orbit but including (or replacing) covariance block with the information extracted from a computed covariance abacus.

To compute an abacus the user shall select, among others:

- Input ephemeris list
- Maximum number of ephemeris considered
- Time span of the estimated and predicted parts of the ephemeris
- Time step for orbital comparisons
- Abacus time span (relative time to OD time)
- RMS threshold
- Fitting rejection threshold
- Minimum covariance values
- Time interval for points association

The computed abacus are shown in tabular format and also in a graphical way (see Figure 4)

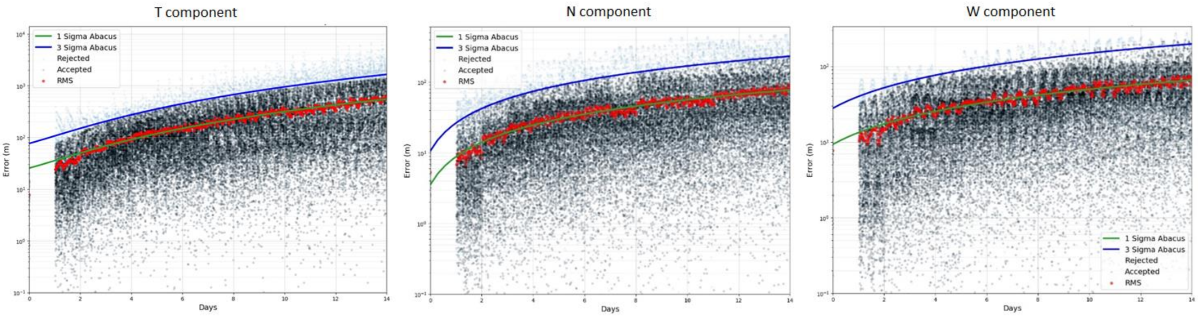

Figure 4 – Covariance abacus graphical result

With the aim of offering a good user experience CONAN has two additional panels for listing and filtering satellite ephemeris and computed covariance abacus respectively.

2.5. CA SERVICE STATISTICS BOARD

All the information imported and generated in CONAN is stored in an internal database. This fact has the advantage of relatively easily computing statistics related to that data. Statistics on the CA Service are a very useful means to measure the service performances (KPIs) or to analyse past data and extract valuable conclusions to improve the service. CONAN has a dedicated panel where multiple statistical parameters can be shown graphically or in tabular format. Once the panel is populated with the chosen plots or tables, a report can be exported into the filesystem.

The following statistics are available:

- **Number of CDMs**. Total number or number of CDMs classified per series according to:
    - CDM source: S3TOC, COO, 18th SPCS, CONAN
    - CDM nature: operational / test
    - CDM type: OPSvsCAT, OPSvsCDM, CDMvsCDM, etc.
    - CDM autonomy
    - Primary satellite
    - Primary constellation
    - Primary's orbital regime
    - Intervals on primary HBR
    - Secondary type: payload, debris
    - Secondary satcat size: SMALL, MEDIUM, LARGE, UNKNOWN
    - Intervals on secondary HBR
    - Risk Level
    - Intervals on Miss Distance
    - Intervals on Radial Distance
    - Intervals on Along-track Distance
    - Intervals on Cross-track Distance
    - Intervals on PoC

    - Intervals on scaled PoC
    - Intervals on Kp
    - Intervals on Ks

- **Number of conjunction events**. Total number or number of events classified per series according to:
    - Originator of its CDMs: S3TOC, French OC, 18thSPCS (one or more)
    - Who has first detected it: S3TOC, French OC, 18thSPCS
    - Event autonomy: partial, complete, detected autonomously or not.
    - Primary satellite
    - Primary constellation
    - Primary's orbital regime
    - Intervals on primary HBR
    - Secondary type: payload, debris
    - Secondary satcat size: SMALL, MEDIUM, LARGE, UNKNOWN.
    - Intervals on secondary HBR
    - Maximum Risk Level reached
    - Intervals on minimum Miss distance reached
    - Intervals on minimum Radial distance reached
    - Intervals on minimum Along-track distance reached
    - Intervals on minimum Cross-track distance reached
    - Intervals on maximum PoC reached
    - Intervals on maximum scaled PoC reached

- CA Service timeliness: **Average time between consecutive CDMs of the same event** and **Time between TCA and first CDM published**. Considering all CDMs or a selection by:
    - CDM source: S3TOC, COO, 18th SPCS, CONAN
    - CDM type: OPSvsCAT, OPSvsCDM, CDMvsCDM, etc.
    - CDM autonomy
    - Conjunction event
    - Risk level
    - Etc.

- **Secondary size**: considering all conjunction events or a selection by:
    - Originator of its CDMs: S3TOC, French OC, 18thSPCS (one or more)
    - Primary satellite
    - Primary constellation
    - Primary's orbital regime
    - Intervals on primary HBR
    - Secondary type: payload, debris
    - Secondary satcat size: SMALL, MEDIUM, LARGE, UNKNOWN.
    - Maximum Risk Level reached

Further statistics can be easily added.

3. DESIGN

CONAN has been designed based on the needs of the users of the tool: O/O users of the EU SST CA service and S3TOC analysts. CONAN is able to operate in two modes: **on-line mode** where it continuously retrieves external data and CDMs, and **off-line mode** where the tool is not connected to external sources, and the data comes only from local disk.

In order to cover the required functionalities both in on-line and off-line modes and also benefit from the advantages of a standalone application and a client-server based application, CONAN has two different deployment versions:

- **CONAN Desktop**: Standalone application that is fully operational by itself but does not act as a service. The application can ingest the external data from local disk or from the external sources but only under user request. CONAN Desktop installation is beforehand customized for each user needs. It provides all the computational functionalities and the database is embedded within the application.

- **CONAN Server**: Service application to be deployed in one place and remotely used by multiple user. It is always running and performs some automated tasks such as periodical external data ingestion or certain processes launch such as the automatic computation of CAM for CDMs with ALERT risk level. CONAN Server and CONAN Desktop only differ in one component that manages the automatic tasks and in the database which in this case is not embedded within the application but running on a database service allowing better performances for higher quantity of data.

Both deployments include user login in order to trace in a log file the user's actions.

3.1. EXTERNAL INTERFACES

The **external interfaces** of the tool are designed to provide the data necessary to ensure CONAN functionalities. It is able to retrieve CA data from the EU SST Service Provision Portal containing the conjunction events information produced by the EU SST CA service provided by the S3TOC and COO. Besides, CONAN is able to retrieve CDMs from the 18th SPCS from SpaceTrack. In order to carry out conjunction analysis CONAN needs EOP and Leap seconds data, which is provided by IERS, and solar flux and geomagnetic activity provided by NOAA. The external interfaces of the tool are depicted in Figure 5.

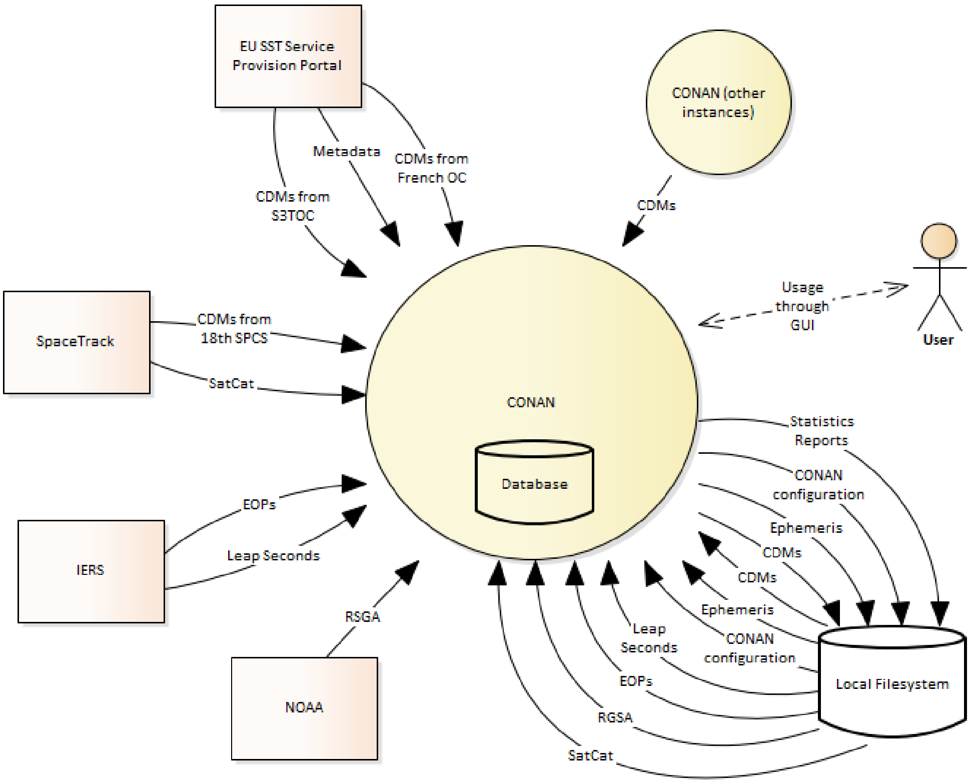

Figure 5 – CONAN external interfaces

3.2. ARCHITECTURE

CONAN software tool is divided in three main logical components and the database, according to the functionality:

- **Frontend** or User Interface: it is the means by which the user and the system interact. It is in charge of displaying all the data contained in the tool. It includes configuration forms, results and graphical information.

- **Backend**: It is the core of the tool in charge of managing user requests, loading external data, managing the database and calling computational modules.

- **Computational Layer**: A set of libraries responsible for providing the flight dynamics functionalities such as CDM analysis, CAM calculation, covariance abacus calculation and statistics calculation.

The next diagram (Figure 6) shows the complete model of the tool with the software components and their relationships.

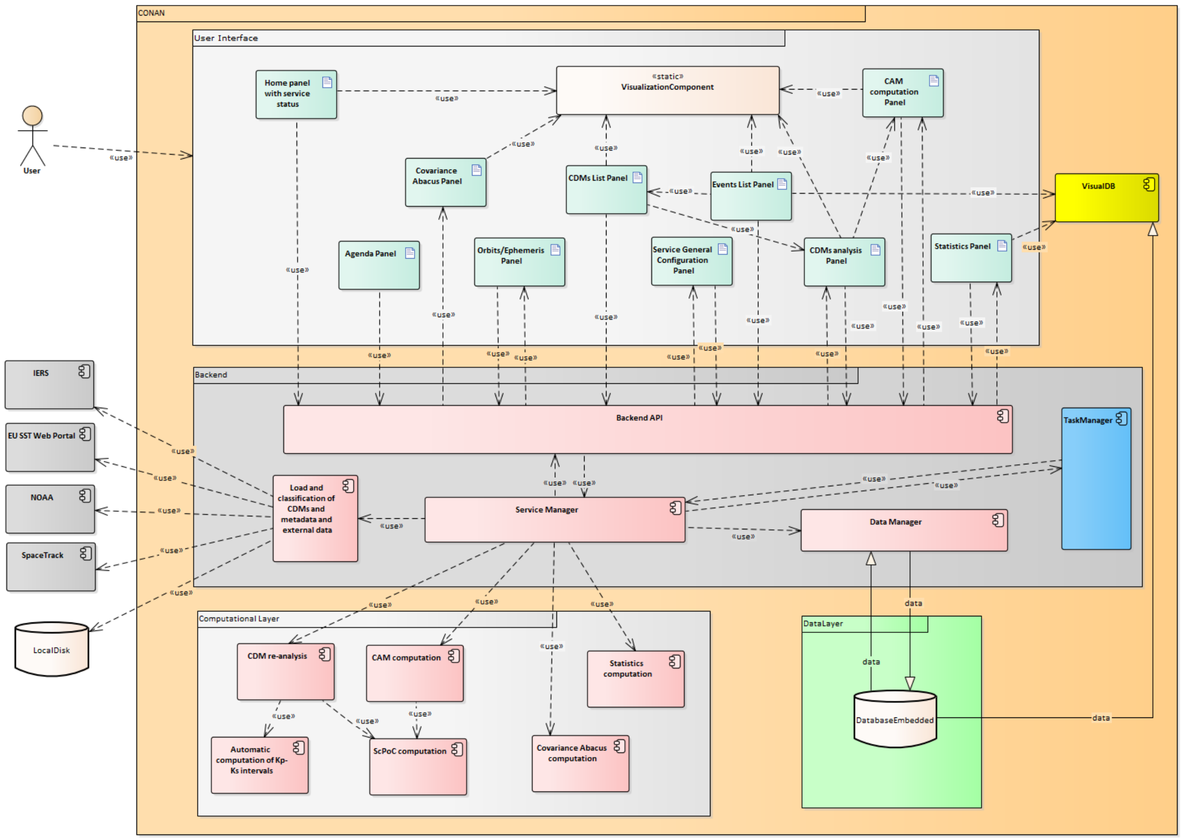

Figure 6 – CONAN SW architecture

Pink coloured boxes are software components of CONAN, grey coloured boxes are external entities or sources of data and blue coloured boxes are components belonging to CONAN Server only. Then soft green coloured boxes are panels of the User Interface which cover functionalities related to visualization and configuration and are considered as software components for design purposes. The yellow box indicates a component to produce graphical plots.

Frontend:

- Home Panel: showing a summary of the CA service status.
- Service General Configuration Panel: containing the satellites and constellations configuration as well as default configuration for some computations.
- Events List Panel: it allows to visualize the list of conjunction events in the database, the CDMs belonging to them and graphical plots related to the evolution of each event.
- CDM List Panel: to specifically visualize the list of CDMs and graphical plots related to each one of them.
- CDM Reanalysis Panel: it allows the user to configure and run an analysis of a CDM.
- CAM Computation Panel: it allows the user to configure and run the computation of CAM solutions.
- Agenda Panel: for visualization of contact data. O/Os would list here the points of contact in the EU SST consortium. OCs would list here the points of contact of the O/Os.

- Ephemeris Panel: to visualize the list of ephemeris in the database for the primary objects that are available to be used for computations.
- Covariance Abacus Panel: it allows the user to configure, run and list the covariance abacuses.
- Statistics Panel: it allows the user to generate statistics and reports.
- Visualization library: it is able to generate the graphics and plots of the tool. It is separated from the definition of the panels because it is a generic library used by several panels to generate the plots.

Backend:

- API: Interface between the backend and the frontend of the tool. It contains the methods that request to the Service Manager for a specific computational process, for external data import, to extract data for visualization or to request a modification in the configuration.
- Service Manager: Software component that acts as the orchestrator of the application. It receives requests from the API and consequently proceeds to call the corresponding module to answer the request. It also transmits/asks data to the Data Manager to be stored/retrieved in/from the Database. In CONAN Server this module contains an additional layer to communicate with the Task Manager.
- Data Manager: Software component than corresponds to the interface between the Database and any other component.
- Load data from external sources: Software component that fulfils the functionality to import data from external sources or the local disk.
- Task Manager: Exclusive component of CONAN Server. It acts as a service requesting to execute tasks such as the automatic import of external data and the CAM computation for CDMs with ALERT Risk Level.

Computational Layer:

- CDM Reanalysis Computation: Software library that carries out the re-analysis of CDMs and calls for scaled PoC computation.
- CAM Computation: Software library that computes CAM solutions for a CDM for an input configuration.
- ScPoC Computation: Software library that implements the functionality of computing the ScPoC.
- Kp-Ks Intervals Computation: Software library that implements the functionality of computing the optimum Kp-Ks intervals from the data of several CDMs for a specific event.
- Statistics Computation: Software library that implements the statistics calculation.
- Covariance Abacuses Computation: Software library that implements the calculus of covariance abacuses for the primary objects.

3.3. DATABASE

Finally, the design of the database has been carried out trying to divide the parameters corresponding to the transitory state of an object belonging to a CDM, and the fixed or configuration parameters.

Figure 7 shows the tables of the database and relationships between them in a MER diagram:

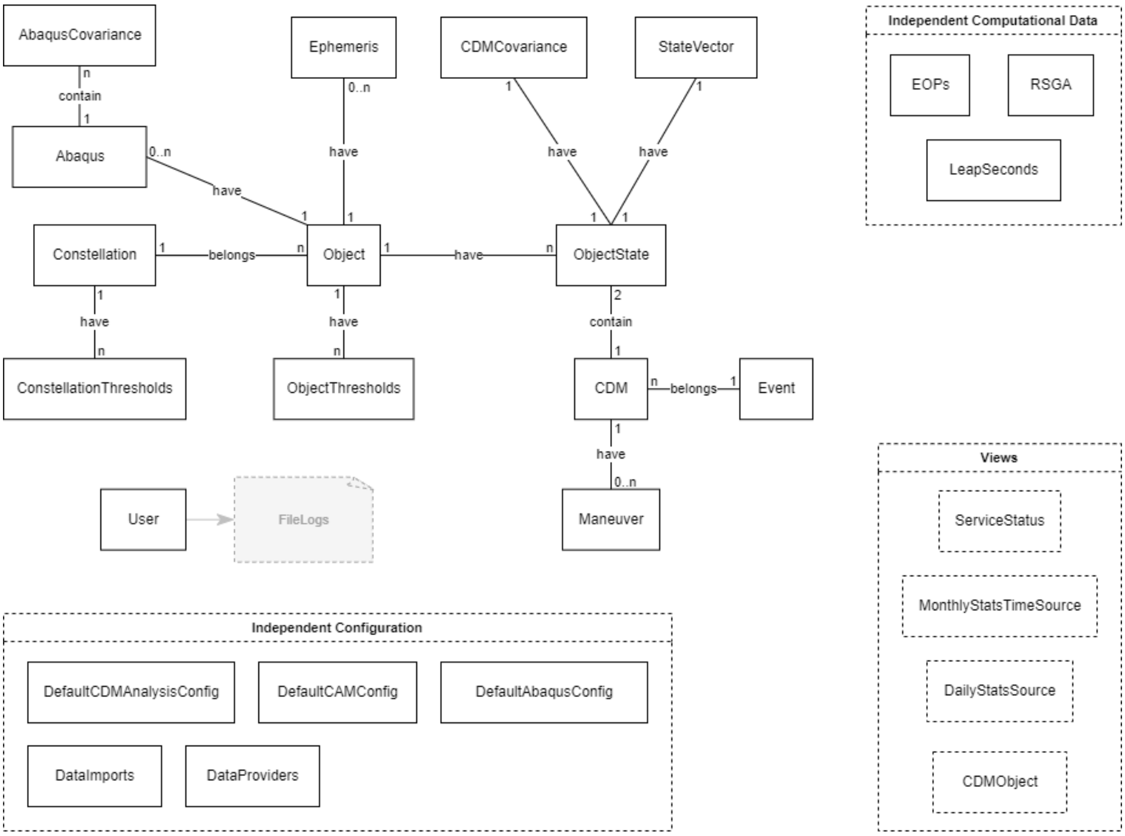

Figure 7 – CONAN database MER diagram

4. ALGORITHMS

This section is intended to describe the algorithms implemented in **CONAN** for conjunction detection, evaluation and prediction of the collision probability and collision avoidance manoeuvre computation.

4.1. CDM RISL LEVEL

The software defines three levels of risk: **INFO, WARNING** (Interest Event) or **ALERT** (High Interest Event). This classification is based on a series of configurable thresholds for the following conjunctions parameters: Screening volume, Miss-distance (MD), the distance by component in the local reference frame (R, T, N), Scaled Probability of Collision (ScPoC) and the Time to TCA (T2TCA).

This set of parameters is configured for each risk level and for each satellite or constellation, according to the user's needs. The algorithm to compute the risk level is as follows:

```
IF { { [ (
            MD < thr_MD
         ) AND (
           (
              volume = CUBOID AND
              R < thr_R AND T < thr_T AND N < thr_N
           ) OR (
              volume = ELIPSOID AND
              sqrt(R^2/thr_R^2 + T^2/thr_T^2 + N^2/thr_N^2)
           )
         )
       ] OR [
         ScPoC > ScPoC
       ]
     } AND {
       T2TCA ≤ threshold_T2TCA
     }
   } THEN {
      Risk level is [ALERT, WARNING].
   } ELSE {
      Risk level is [WARNING, INFO].
   }
```

Equation 1

The risk level is analysed from higher to lower risk level: First, the risk level is calculated based on this algorithm, using the HIE thresholds. If the CDM complies is categorised as HIE in this step, otherwise it is checked against the IE thresholds. If it complies it is categorised as WARNING, otherwise, it is classified as INFO.

CONAN launches this process in background when **importing new CDMs** without a risk level assigned (from 18th SPCS or loaded from the local filesystem) and keeps the risk level coming from the CDMs downloaded from the EU SST Service Provision Portal.

4.2. PROBABILITY OF COLLISION

The algorithms for calculating the **probability of collision (PoC)** make possible to assess the risk of a given event in terms of probability, taking into account the relative position between both objects, their size and the uncertainty with which their position is known. The complexity associated with the calculation of this probability depends on the **relative velocity of the encounter**.

There are **assumptions that are common** to most methods developed so far for collision probability computation:

- The collision probability is derived for a **linear encounter** where the relative motion of the two objects can be considered linear (i.e., large and constant relative velocity) [REF. 1]. This hypothesis is valid for most conjunctions, especially in the LEO orbital regime, generally with high relative velocities. In the GEO orbital regime, it is possible to have long encounters and it becomes necessary to account for **nonlinear effects.**

- The **position uncertainty** can be represented by a **Gaussian distribution** characterised by the covariance of the orbital state of the objects. This is consistent with orbit determination and covariance propagation techniques. It is a reasonable assumption when the propagation models are accurate and do not introduce systematic errors.
- There is **no uncertainty in the velocity** during the encounter. It is much smaller than that of the position covariance both for short and long encounters and can be neglected.
- There is **no correlation between the orbital states** of both objects since their orbits are estimated independently and thus the covariance of the relative position vector can be computed by adding the position covariance of both objects at any time [REF. 2].
- The **objects are considered as spherical objects** with a diameter equal to the largest dimension.

For the purposes of the derivation, it is more convenient to consider the **relative motion** of the primary object with respect to the secondary object. Moreover, it is assumed that the **secondary is a point mass** with its position uncertainty represented by the **addition of the individual covariance** of both objects while the primary is represented as a sphere with a diameter equal to the addition of the largest dimension of each object.

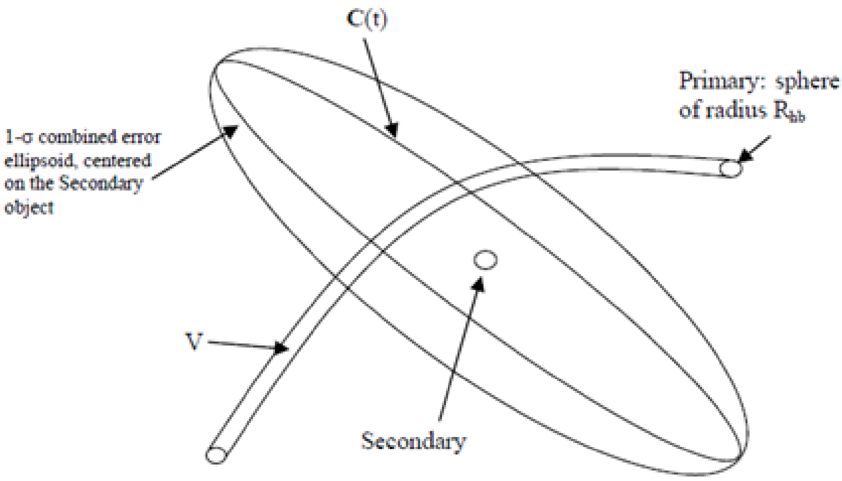

Figure 8 – Relative motion during the close conjunction

Short encounters

In events with **high relative velocity**, or short encounters it can be assumed that:

- The motion of both objects can be represented by a rectilinear motion
- The position uncertainty during the encounter is constant

This allows to define a conjunction plane (B-Plane) perpendicular to the relative velocity vector in the TCA and to project on this plane the position of both objects, their size (Hard-Body Radius, HBR) and the combined error ellipsoid to calculate the probability of collision. The algorithm used by **CONAN** in high relative velocity events is the one proposed by **Akella-Alfriend** in [REF. 3], which calculates the PoC as a surface integral over the HBR of the Gaussian distribution associated with the projection of the combined error ellipsoid on the B-Plane.

$$P_c = \iint \frac{1}{(2\pi)^{2/3}|C|^{1/2}} \exp\left(-\frac{(\vec{x}-\vec{x}_0)C^{-1}(\vec{x}-\vec{x}_0)}{2}\right) dA$$

Equation 2

where x-x0 is the relative position of the secondary object to the primary object in the B-Plane and C is the combined position covariance matrix in the B-Plane.

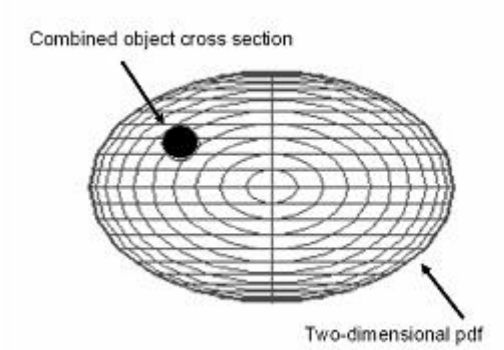

Figure 9 – 2D probability distribution function and target footprint in the B plane

Long encounters

In events where the **relative velocity is sufficiently low** (typically, below 10 m/s), the previous method loses accuracy by not taking into account non-linear effects in the relative motion between the two objects. In these cases, the collision probability has to be evaluated as the cumulative probability over the volume swept *V* by the primary object in its relative motion around the secondary object and the error ellipsoid. All these non-linear methods are based on expressing the **time integral yielding the collision probability as a three-dimensional integral** over the volume swept by the primary in the n-σ covariance ellipsoid of the secondary. The volume is typically a curvilinear cylinder, which makes the computation a bit complicated. Depending on the level of accuracy and the level of non-linearity of the relative motion, the methods to compute the volume integral differ in terms of complexity.

$$P_c = \iiint \frac{1}{(2\pi)^{2/3}|C(t)|^{1/2}} \exp\left(-\frac{(\vec{x}-\vec{x}_0(t))C(t)^{-1}(\vec{x}-\vec{x}_0(t))}{2}\right) d\Omega$$

Equation 3

Different approaches are available in the bibliography:

- **Patera** proposes in [REF. 4] **a method based on dividing the curvilinear tube volume into smaller tube sections** where the linear approximations for the relative velocity holds and the covariance can be assumed to be constant.

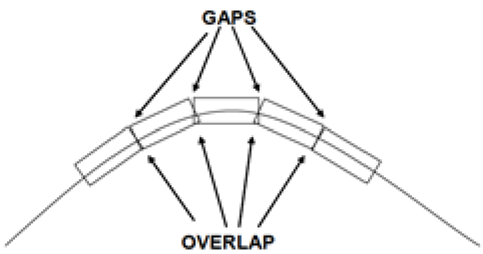

Figure 10 – Tube sections for non-linear relative motion. Gaps and overlaps are visible (from [REF. 4])

- **Chan** proposes a similar method in [REF. 5].

- **Alfano** proposes several different methods of increasing complexity: **adjoining cylinders method** and **bundled parallelepipeds method**, which deal with gaps and overlaps between the smaller tubes (see [REF. 6]) and **voxels method,** consisting of a complete three-dimensional numerical integration of the swept volume with a transformation to a Mahalanobis space at discrete times, very time consuming (see [REF. 7]).

- A similar idea to the first one from Alfano is used by **McKinley** in [REF. 8] for the conjunction prediction system used at NASA GSFC.

NASA CARA uses an alternative formulation [REF. 9], based on Coppola's method, which consists of the integral over the conjunction time of:

$$P_c = \int_{t_0}^{t_0+T} R_c(t)\, dt$$

Equation 4

Where Rc, the probability rate, is the evolution of the probability of collision over time and is defined as:

$$R_c(t) = \frac{dP_c}{dt} = \oint_{4\pi} I(\hat{\mathbf{r}}, t)\, d^2\hat{\mathbf{r}}$$

Equation 5

Where I is a map between points on the unit sphere and time, but also depends on the dynamical model quantities x(t) (Object state vector with time) and P(t) (Object position uncertainty with time). To integrate the time integral, CARA uses a numerical method, the trapezoidal integration, while the Rc integral is solved using a numerical integration over the Unit Sphere, using the Lebedev quadrature scheme with an algebraic order of 131. The covariance evolution is estimated with a simple covariance propagation using Keplerian Two-Body motion.

CONAN, uses the **Coppola** for **long encounters,** including last improvements presented by Hall (see [REF. 10]).

The **algorithm selection** can be done either **manually** or **automatically** depending on a configurable threshold for the relative velocity.

4.3. SCALED PROBABILITY OF COLLISION

The collision probability is very sensitive to changes in the sizes of the covariance matrices of the objects. In addition, the covariance matrices nominally assigned to each object may not be sufficiently realistic and it may be either pessimistic (if the jumps experienced by the object's position vector are much smaller than its position uncertainty) or optimistic (if the jumps experienced by the position vector escape the position uncertainty region).

In both cases, the lack of realism in the covariance matrices leads to errors in the estimation of the real risk of the event.

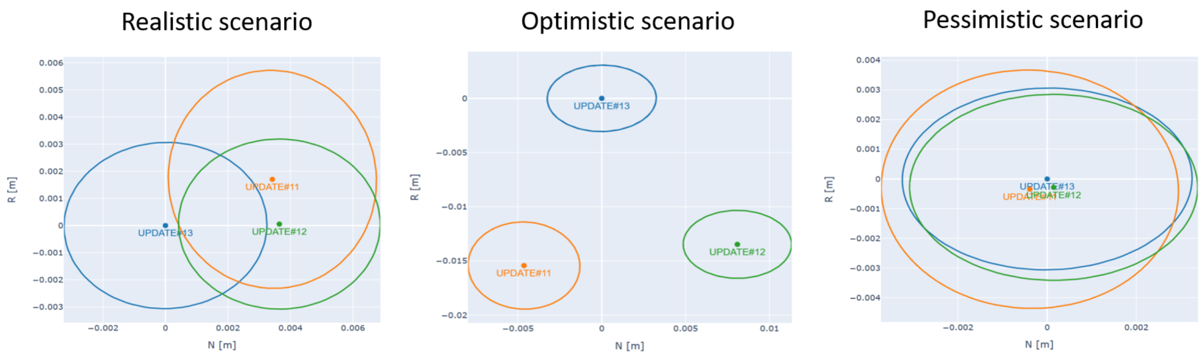

Figure 11 – Visualization of orbital positions and successive covariance updates

To solve this problem we use the **scaled probability of collision (ScPoC)**, defined as the probability of collision obtained by scaling the covariance matrices of the two objects of the form:

$$C = K_p^2 C_p + K_s^2 C_s$$

Equation 6

where C is the combined position covariance matrix and Kp and Ks are two scaling factors that apply to the standard deviations of the covariance matrices of both objects.

The scaled probability algorithm defines ranges for the factors Kp and Ks (the range [0.25, 4] is typically used) over which the maximum probability value is sought. This maximum value is considered as the scaled collision probability.

CONAN computes the Scaled probability of collision by default for all CDMs reanalysed. For the CDMs imported in the tool, it is read if present in the CDM or its associated metadata. It is worth to say that the **b-plane plot** in the CDMs List Panel of CONAN allows to simultaneously represent multiple CDMs to allow understanding the realism of covariance (similarly to the cases presented in Figure 11).

4.4. AUTOMATIC CALCULATION OF KP AND KS RANGES

The main problem with the use of the scaled collision probability is the decision of which ranges to consider for Kp and Ks. Using a very conservative range could lead to an overestimation of risk, while an overly optimistic range could lead to an underestimation of risk.

The methodology proposed in [REF. 11] aims to derive proper Kp and Ks ranges for each conjunction event based on previous information of the event that is available at the time of analysis. This methodology computes the **Mahalanobis distances** between the latest predicted position of some object at the TCA and its predicted positions from previous independent updates (associated with previous and different orbit determinations). This distance is defined as:

$$D_M(\vec{x_i}) = \sqrt{(\vec{x_i}-\vec{x_j})\,(C_i+C_j)^{-1}(\vec{x_i}-\vec{x_j})} \qquad i > j$$

Equation 7

Since the errors in position should follow a **3D zero-mean Gaussian distribution**. As a consequence, the **square of the Mahalanobis distance** will be expected to follow a **Chi-squared distribution** of 3 degrees of freedom if the assumption related to the position errors is true. Thus, an observed cumulative distribution function (CDF) can be obtained by sorting in ascending order the Mahalanobis distances computed for the latest update of the object with respect to the previous ones. By comparing the observed CDF against the theoretical Chi-squared distribution, a range of scale factors K for the standard deviations can be computed to improve the consistency between both CDFs

To deduce the range of scale factors from the comparison between the observed and the theoretical distribution, statistical methods such as the **K-Point** or the **K-Interval** can be used, which try to adjust independently each element of the observed distribution to the theoretical one and from there deduce the range of scale factors sought. Goodness-of-fit methods can also be used, such as the **Kolmogorov-Smirnov test** (see [REF. 12]), which attempts to fit all the elements at the same time. Typically, the union of the results of the K-Interval method and the Kolmogorov-Smirnov test is used as the final solution, to ensure that the original covariance value is included in the analysis.

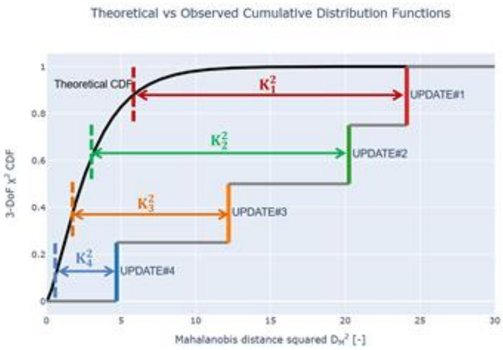

Figure 12 – K-Point coefficient method

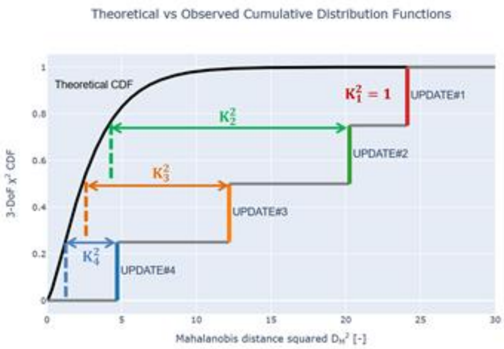

Figure 13 – K-Interval coefficient method

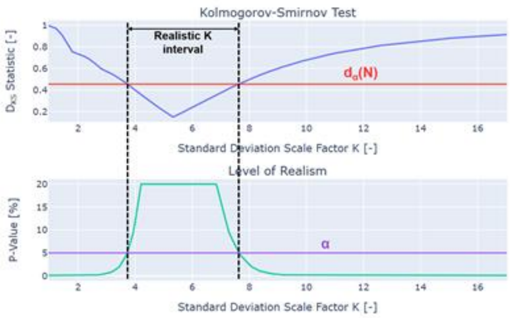

Figure 14 – Kolmogorov-Smirnov test

In **CONAN,** the user can select whether to use a fixed value of Kp and/or Ks, whether a fix interval, or the option of automatic computation of the interval using any of the three available methods.

4.5. THEORY OF PREDICTION OF THE EVOLUTION OF COLLISION PROBABILITY

The **prediction of the evolution of the probability of collision** aims to anticipate how the risk of the event will evolve in future updates. This prediction can be very useful in high-risk events, since it could show when it is not necessary to perform a manoeuvre (if the risk is expected to disappear as we approach the TCA) or when it will be necessary to manoeuvre (if the risk is expected to remain or increase as we approach the TCA).

The proposed algorithm to perform this prediction is similar to the one proposed by Eumetsat in [REF. 13] and is based on using the information available in the last update of the event (relative position and combined covariance matrix in the B-Plane) to generate a set of possible relative positions in the B-Plane in the future. This set of relative positions can be obtained from generating random positions within the 3σ error ellipse or from generating a weighted mesh with its probability in the B-Plane that includes this ellipse.

For achieving this prediction of the collision risk evolution, a prediction of how the covariance matrices of the objects will evolve must be performed first. Fitting functions together with the least-squares method, are used to model the reduction of the uncertainties in position in an RTN orbital reference frame. In order to achieve an accurate prediction for the covariance matrix, the objects updates shall be independent (different orbit determinations) otherwise the latest available covariance matrix is used as the predicted one (conservative solution).

Once the covariance matrices of both objects have been predicted, the methodology generates a set of feasible miss vectors in the B-Plane in the future based on the data of the latest update of the event.

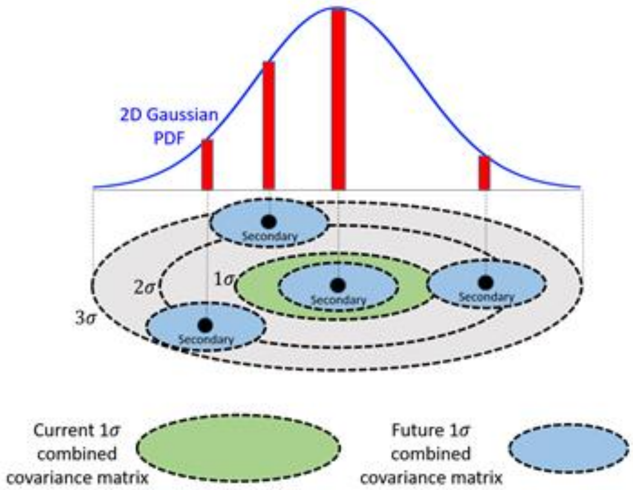

Figure 15 – Set of feasible miss vectors in the future and predicted covariance (blue ellipse)

In order to compute the distribution of future PoC, a weighted scattering is carried out. This approach is based on meshing the region of the B-Plane that contains the 3σ error ellipse defined by the latest combined covariance matrix, as shown in Figure 16. In that way, if the latest combined covariance matrix is realistic enough, it is ensured that most of the feasible future miss vectors will be considered. However, all the points of the mesh are not equally likely. Therefore, these points must be weighted to take into account the probability of getting such miss vectors in the future. The weighting factors can be obtained from the 2D Gaussian probability density function associated with the latest combined covariance matrix in the B-Plane.

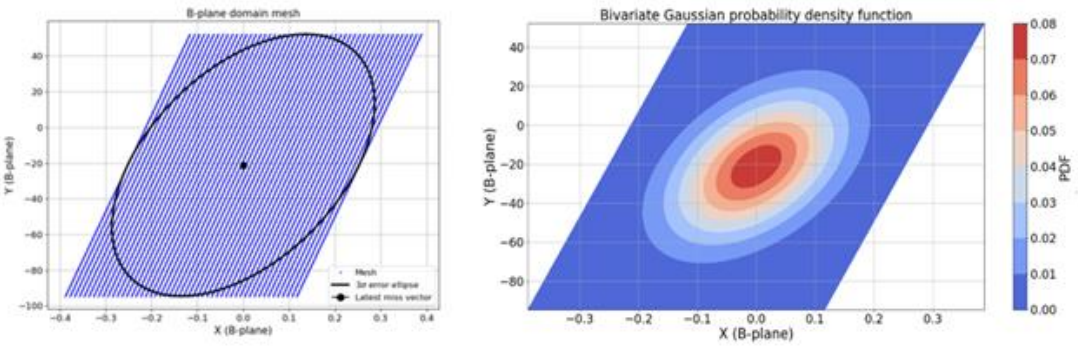

Figure 16 – Weighted scattering approach to generate the set of feasible miss vectors in the future

Once the set of feasible miss vectors in the future has been generated, it can be transformed into a population of feasible collision probabilities in the future (Figure 17) by means of using the predicted covariance matrices. This population can be used to estimate the probability of getting a future collision probability above a threshold defined during the analysis. This probability gives an idea of how the risk of collision will evolve in the future.

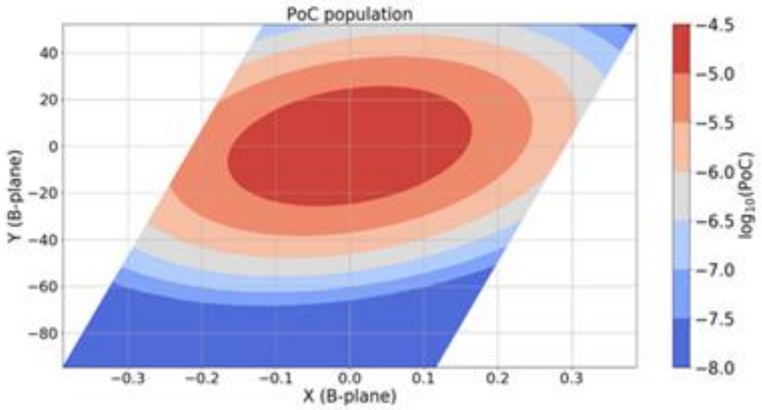

Figure 17 – Predicted collision probability population

With all this information it is possible to estimate the probability that the future PoC or ScPoC will be above a certain threshold in the future, as shown in Figure 18.

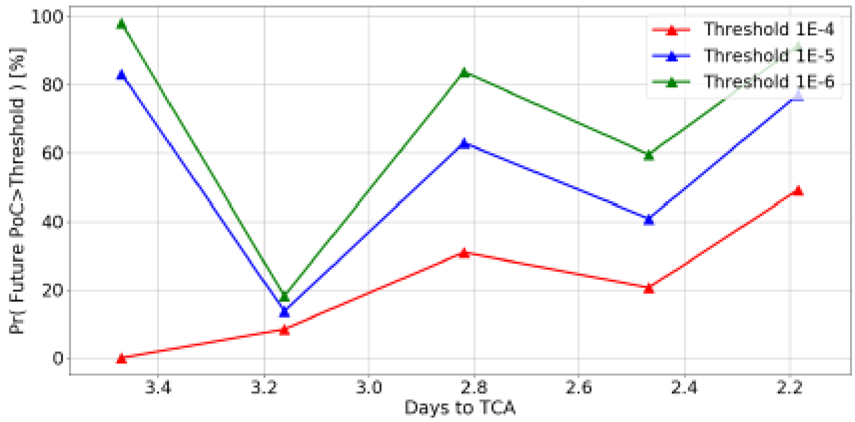

Figure 18 – Probability of being above a given PoC threshold in the future

The **CONAN** user can manually launch the computation of the evolution of collision probability for a specific CA Event, from the Events List Panel.

4.6. COLLISION AVOIDANCE MANEUVER RECOMMENDATION

The following are the algorithms related to manoeuvre computation implemented in CONAN:

Optimal CAM

In those events where the risk is too high, either for reasons of collision probability or for reasons of geometry (relative position), it is necessary to evaluate the impact that an avoidance manoeuvre would have on the encounter. In those cases where there is no constraint associated with the instant of the manoeuvre or its direction, the algorithm allows calculating the optimal 3D manoeuvre that reduces the collision risk the most. This is done by using the PoC-gradient method along the three directions at the point of the orbit in which the manoeuvre is to be performed. It is possible to compute this optimal manoeuvre for a specific epoch or define a certain range of epochs before the TCA in order to look for the best candidate between those computed. The best one will be the closest manoeuvre to the TCA that fulfils the post-CAM probability threshold and the input manoeuvre constraints defined by the user.

Parametrical CAM analysis

For the realistic cases, in which there are constraints related to the epoch and the direction of the manoeuvre. The algorithm implemented can perform a parametric analysis where several ΔV values can be analysed at different epochs (both in the positive and the negative along-track direction). The epochs to be analysed are obtained from the expression:

$$[t_{CAM,0}, \dots, t_{CAM,n}] \models TCA - \left(\frac{[N_{min}, \dots, N_{max}]}{p} + offset\right) \cdot T$$

Equation 8

where N_{min} and N_{max} are a minimum/maximum integer number of orbits, p is the integer number of points in which one orbit will be divided, offset is a parameter used to define the initial point from which the epochs are computed and T is the orbital period. The software can propose a manoeuvre recommendation (best candidate) between all the manoeuvres computed during the parametrical analysis. The best solution will be the closest manoeuvre to the TCA that fulfils all post-CAM thresholds (miss-distance, radial distance and collision probability) and the input manoeuvre constraints defined in the configuration panel. In case that no suitable candidate is found, the software can extend the original range of epochs in order to try to find a manoeuvre recommendation.

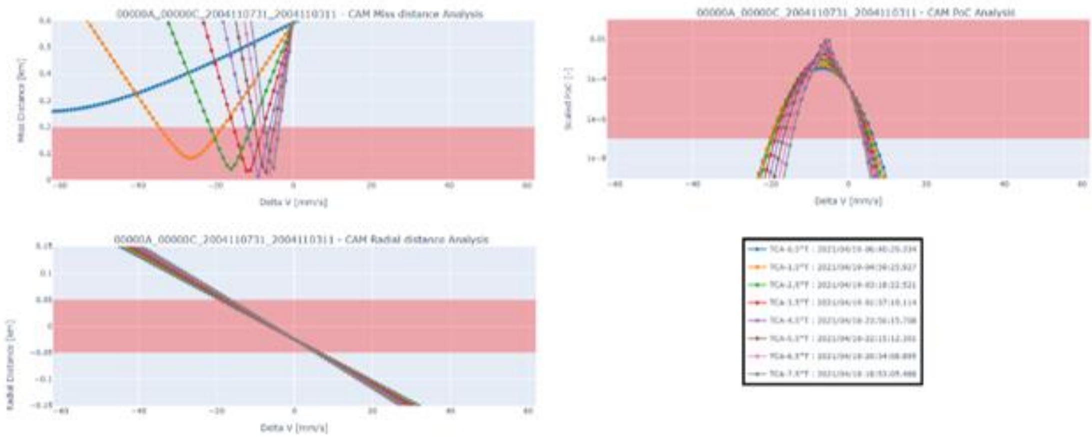

Figure 19 – Dynamic graph generated by the software for collision avoidance manoeuvre calculation

Conjunction re-analysis

For the re-analysis of the conjunction event taking into account the computed collision avoidance manoeuvres, the methodology implemented propagates the deviation caused by the manoeuvre with respect to the nominal orbit of the satellite, keeping fixed the nominal orbit. Since the ΔV is usually small, it is reasonable to assume that the post-manoeuvre can be computed as a small and linear correction of the original orbit. To this end, the state transition matrix propagated over the satellite orbit is used. The correction to be applied to the original orbit can be expressed as:

$$\bar{X}\,(t) = \bar{X}_0(t) + M(t) \cdot d\bar{X}$$

Equation 9

where X is the state vector at the time t after the manoeuvre epoch, X_0 is the state vector at the same time for the original orbit, M is the state transition matrix at the time t after the manoeuvre epoch and dX is the manoeuvre vector.

Once the post-manoeuvre orbit is computed, the algorithm re-computes the new TCA, relative position and collision probability. For the computation of the new relative position, the post-manoeuvre orbit is used for the satellite (target) and the state vector of the chaser at the new TCA are considered. During the computation of the new collision probability, the impact of the manoeuvre uncertainty must be taken into account. In addition, the pair of Kp-Ks values for which the scaled collision probability is reached are re-computed too.

Post-Screening analysis

The software can also verify that the proposed manoeuvre is not increasing the risk or causing new HIE in the near future. For these ad-hoc analyses, the proposed algorithm consists of taking the orbit that includes the tentative manoeuvre and recovering the CDMs associated with the events in which the primary satellite is involved and that are active at the time of the analysis. In this way, it is possible to measure the impact of the manoeuvre on the event of interest, as well as to make sure that the manoeuvre does not have a negative impact on the rest of the active events. **CONAN** allows to launch this analysis after computing a CAM solution in an easy way.

Manoeuvre Plan Modification

There may be cases where the operator is more interested in modifying the manoeuvre planning instead of performing a specific collision avoidance manoeuvre. For these cases, the software allows to manually introduce one or multiple manoeuvres that will be added to the reference orbit and used to reanalyse one or more CDMs. The user can simulate the effect of changes in the manoeuvre plan and get to the final solution in an iterative manual process.

4.7. REDUCTION BY GLOBAL RISK MANOEUVERS IN CASES OF MULTIPLE CONJUNCTIONS

Nowadays, in S3TOC operations, situations in which several collision events occur at the same time are becoming more and more frequent. That is, there are two (or, eventually, more) high-risk potential collision events that, therefore, require a coordinated mitigation action. In single events it is usual to perform a collision avoidance manoeuvre an integer number of orbital half-periods before the TCA of the event (Parametrical CAM analysis from previous section), being this number odd or even depending on whether a separation in the radial direction or in the along-track direction is intended respectively.

In the case of multiple events, this strategy is not possible since there are two different TCAs and they are usually not such that the possible manoeuvre times that would be deduced from each of them are similar. In these situations, a manoeuvring time optimization can be performed with the aim of **reducing the collision probability and/or increasing the distance in the TCAs of both collisions** within acceptable limits in unison. In this optimization it is expected that there will be several local minima, so a parametric analysis has to be done followed by a local refinement by means of optimization. In this optimization process, the manoeuvre hypothesis will be made in the tangential direction (i.e. along-track) but nevertheless, the manoeuvre size has to be optimized together with the manoeuvre time in order to calculate manoeuvre time and manoeuvre size that results in the smallest manoeuvre size that allows both collision events to be taken out of the risk zone, both in ScPoC and in distance at the TCA.

CONAN will allow to compute a CAM taking as input more than one CDM in order to use this algorithm.

4.8. COVARIANCE ABACUS COMPUTATION

In some cases, the collision probability cannot be calculated because the covariance of one of the conjunction objects is not available. In these cases, it is useful to be able to apply an "observed" covariance from the differences between successive, uncoupled orbital determinations. The covariance abacus algorithm allows calculating the "observed" covariance of a given object, the primary for which the O/O has no covariance information.

The proposed algorithm performs a comparison between successive orbits of the object to provide an estimate of the evolution of the orbital uncertainty of the object with time.

A "determined orbit" is one whose duration covers the measurements used in the orbit determination process, and a "predicted orbit" is one that results from propagating a given orbit into the future. Determined orbits are more accurate than predicted orbits and are considered "true" orbits.

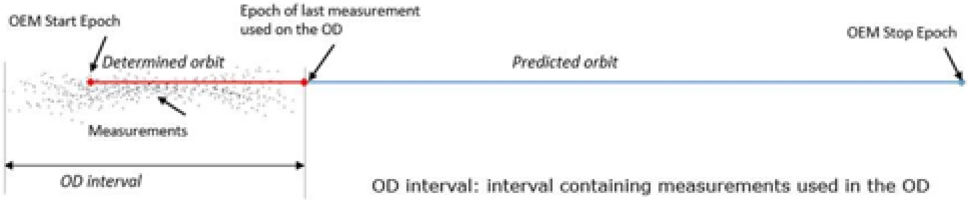

Figure 20 – Determined orbit and predicted orbit

To calculate the covariance abacus, the predicted orbits available at a given time period are compared with subsequently determined orbits and their differences as a function of time are calculated.

When the number of orbits compared is sufficiently large, this process leads to obtaining a statistical dispersion of the orbital differences as a function of time.

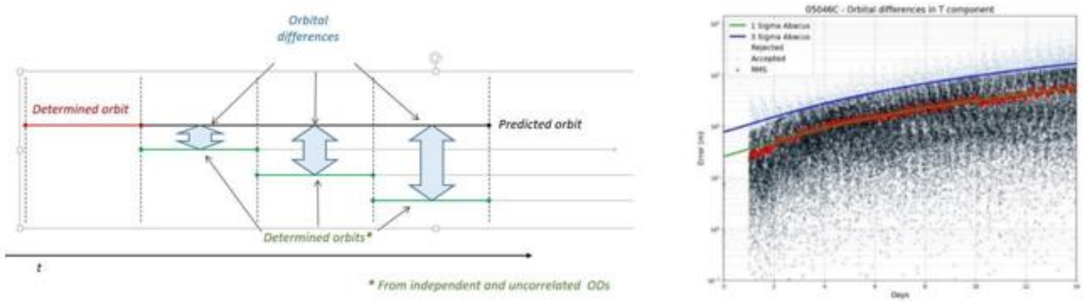

Figure 21 – Differences between determined and predicted orbits (Left) and Example of covariance abacus (Right)

The process consists of the following:

1. Least squares adjustment of differences to a polynomial of second degree.
2. Detection and discarding of outliers.
3. Calculation of the RMS of the accepted differences by small time interval.
4. Least squares fit of the RMS points to a second degree polynomial.

Performing this process independently in each of the spatial directions, the value of the variance of the position in each direction is obtained and from there a diagonal covariance matrix is generated and applied to each step of the orbit.

5. CONCLUSIONS

The **CON**junctions **AN**alysis Tool, **CONAN**, represents a step forward in the provision of CA Service by the EU SST to O/O subscribed to S3TOC as CA Service Providers, offering new functionalities or improving performances and response times related to functionalities already in place, both for S3TOC analysts and for O/O operators.

Agglutinate conjunction events information, reanalyse CDMs with different configurations, computing CAMs for one or multiple simultaneous encounters, computing covariance abacus to allow computation of probability of collision, and furnishing interesting and fully configurable statistics reports are the main functionalities of the software.

The design is presented including the external interfaces, the software architecture and a description of the internal database.

Finally the algorithms chosen for the different computations within CONAN are described and justified. All of them are in line with the current methodologies implemented in S3TOC.

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